

Capacity of Compound MIMO Channels with Additive Uncertainty

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Abstract

This paper considers reliable communications over a multiple-input multiple-output (MIMO) Gaussian channel with an additive channel uncertainty unknown to both the transmitter and the receiver — which is referred to as a compound channel in the information theory literature. We derive a closed-form expression of the capacity of the compound MIMO channel with a bounded channel uncertainty model. By proving a saddle-point property, we show that the capacity of the compound MIMO channel is equal to the minimum capacity of all the possible channel realizations. This proves a conjecture of Loyka and Charalambous (*IEEE Trans. Inf. Theory*, vol. 58, no. 4, pp. 2048-2063, 2012). The technical difficulty in showing this saddle-point property is that the mutual information is non-convex with respect to the channel uncertainty. We resolve this difficulty by establishing a novel matrix determinant inequality. A lower bound on the outage capacity of MIMO channels is also provided by using this capacity result.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) technique has been widely employed as a revolutionary technique to improve the spectral efficiencies of wireless networks. The performance of MIMO communications relies heavily on access to the channel state information (CSI). When the CSI is perfectly known at both the transmitter and receiver, the MIMO channel can be converted to multiple parallel single-input single-output channels through a singular value decomposition of the channel matrix [1]. However, in practice, the transmitter often has some channel uncertainty, due to issues such as finite-length training and finite-rate feedback. The channel uncertainty at the transmitter can result in a significant rate loss, if not taken into consideration in the transmit covariance design.

This important channel uncertainty issue has motivated two categories of research efforts towards reliable communications over MIMO Gaussian channels with channel uncertainty. The first category focuses on stochastic models of channel uncertainty, where the transmitter has access to only the statistics of the CSI, but not to the accurate CSI value. When the channel states change fast over time, the capacity of the channel is described by the ergodic capacity, e.g., [1], [2], [3], [4]. On the other hand, when the channel states vary slowly, the achievable rate is characterized by the outage capacity, which is the maximum data rate such that the outage probability of the mutual information is no larger than a specified value, e.g., [1], [3], [4], [5], [6], [7].

The second category of studies were centered on deterministic models of channel uncertainty, where the CSI is a deterministic variable within a known set, but its actual value is unknown to both the transmitter and the receiver. Such a model is called a *compound channel* in the content of information theory, and its capacity is determined as the maximum of the worst-case mutual information in the set of possible channel realizations [8]. From practical viewpoint, it is the maximum data rate that can be reliably transmitted over *any* channel from the given set. It is important to emphasize that the channel uncertainty at the receiver is not essential in the compound channel, because providing the accurate CSI value to the receiver can not improve the capacity [9], [10].

Due to its importance, the capacity of compound MIMO channels have received considerable research attentions. In [11], the authors investigated a compound MIMO channel model where the set of possible CSI values is isotropy. They showed that the optimal transmit covariance matrix is a multiple of the identity matrix. In closed-loop MIMO systems with feedbacks, the transmitter is able to obtain some knowledge about the CSI, which corresponds to a non-isotropy compound channel model. In this case, the channel is typically modeled as the sum of a known nominal channel and an unknown channel uncertainty

belonging to a given set. The additive channel uncertainty model is able to characterize the error caused by channel estimation and feedback quantization. It has been widely utilized both in the information theoretical studies, e.g., [12], [13], [14], and in the robust algorithm designs of signal processing, e.g., [15], [16], [17], [18]. In [19], the capacity of compound Rician MIMO channels with additive channel uncertainty was studied, where the analysis was restricted to a rank one nominal channel. Arbitrary rank nominal channel was considered in [20], where the channel uncertainty is limited to the singular value of the nominal channel with no uncertainty on the singular vectors, which is unlikely to occur in practice. In [14], the capacity of compound MIMO channels with additive channel uncertainty was derived in some special cases, such as high signal-to-noise ratio (SNR) limit, low SNR limit, and rank two nominal channel. A conjecture was made in [14] for the general capacity expression of the compound MIMO channel with additive channel uncertainty. The capacity of compound MIMO channel with a multiplicative channel uncertainty was obtained in [14], where the multiplicative channel uncertainty may be caused by mobility or calibration inaccuracy.

In this paper, we derive the capacity of compound MIMO channel with additive channel uncertainty for the general scenario. By proving a saddle-point property, we show that the capacity of the compound MIMO channel is equal to the minimal capacity of all the possible channel realizations. The technical difficulty in showing this saddle-point property is that the mutual information is non-convex with respect to the channel uncertainty. In the literature, similar non-convex problems have been resolved by means of convex reformulation [19] and singular value inequalities [14]. However, these methods are not effective for the problem at hand. To that end, we establish a novel matrix determinant inequality, which, together with Theorem 3 in [14], yields closed-form expressions for the capacity of compound MIMO channels with additive channel uncertainty and the universal transmit covariance matrix achieving the capacity. This proves the conjecture proposed in [14]. Finally, by using the relationship between the capacity of compound channel and the outage capacity [19], we derive a easy-to-compute lower bound for the outage capacity of MIMO channels.

The paper is organized as follows. We begin in Section II by providing our channel model. Some preliminary results about the compound MIMO channels is given in Section III. Next in Section IV, we provide our main result in Theorem 2 and discuss its extension and engineering value. A lower bound for the outage capacity of MIMO channels is derived in Section V by using Theorem 2. Finally, conclusion is drawn in Section VI.

A. Notations

The following notations are used. Boldface upper-case letters denote matrices, boldface lower-case letters denote column vectors, and standard lower case letters denote scalars. $\mathbb{R}^{m \times n}$ and $\mathbb{C}^{m \times n}$ denote the set of $m \times n$ matrices with real- and complex-valued entries, respectively. \mathbb{S}_+^n represents the set of $n \times n$ Hermitian positive semidefinite matrices. $\mathbf{X}(S)$ denotes the submatrix of \mathbf{X} , which is obtained by deleting the rows and columns complementary to those indicated by S from the matrices \mathbf{X} . $\text{diag}(x_1, x_2, \dots, x_t)$ denotes a diagonal matrix with diagonal entries given by x_1, x_2, \dots, x_t . \mathbf{I}_t denotes a $t \times t$ identity matrix. $\mathbf{E}_{ij} \in \mathbb{R}^{m \times n}$ denote the matrix whose (i, j) th entry is 1 and all of whose remaining entries are 0. $\mathbf{0}$ represents zero matrix. By $\mathbf{X} \succeq \mathbf{0}$ or $\mathbf{X} \succ \mathbf{0}$, we mean that \mathbf{X} is a Hermitian positive semidefinite or definite matrix, respectively, and $\mathbf{X} \succeq \mathbf{Y}$ means that $\mathbf{X} - \mathbf{Y} \succeq \mathbf{0}$. The operators $(\cdot)^H$, $(\cdot)^{-1}$, $\text{Tr}(\cdot)$ and $\det(\cdot)$ on matrices denote the Hermitian, inverse, trace and determinant operations, respectively. $\sigma_i(\mathbf{A})$ and $\lambda_i(\mathbf{A})$ represent the singular value and eigenvalue of \mathbf{A} , respectively. Both the singular values and eigenvalues are listed in descending order. The spectrum norm of a matrix \mathbf{A} is defined by

$$\|\mathbf{A}\|_2 \triangleq \max_{\|\mathbf{x}\|_2 \leq 1} \|\mathbf{A}\mathbf{x}\|_2 = \sigma_1(\mathbf{A}).$$

$|S|$ denotes the cardinality of the set S , and $S \setminus T$ denotes the set $\{x : x \in S, x \notin T\}$.

II. COMPOUND MIMO CHANNEL MODEL

Consider the complex-valued vector channel:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \quad (1)$$

where \mathbf{y} is a length r received vector, \mathbf{H} is a $r \times t$ channel matrix, \mathbf{x} is a length t transmitted vector with zero mean and covariance $E\{\mathbf{x}\mathbf{x}^H\} = \mathbf{Q}$, and \mathbf{n} is a complex Gaussian noise vector with zero-mean and covariance $E\{\mathbf{n}\mathbf{n}^H\} = \mathbf{I}_r$. We model the MIMO channel \mathbf{H} as an unknown deterministic matrix expressed as

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{\Delta}, \quad (2)$$

where \mathbf{H}_0 is the nominal channel known to both the transmitter and receiver, $\mathbf{\Delta}$ is the channel uncertainty which belongs to a compact and bounded set \mathcal{E} defined by [14]

$$\mathcal{E} \triangleq \{\mathbf{\Delta} : \|\mathbf{\Delta}\|_2 \leq \varepsilon\}. \quad (3)$$

In practice, the region of channel uncertainty $\mathbf{\Delta}$ may have an irregular shape. In this case, we can find some ε to obtain a larger and conservative region \mathcal{E} expressed in (3). In the stochastic channel uncertainty

model, the possible region of Δ may be unbounded, which is quite different with the deterministic model in (3). In this case, we obtain a lower bound for the outage capacity of MIMO channels; see Section V for more details.

The actual realization of Δ is not available at either the transmitter or the receiver. The power constraint at the transmitter is given by $\mathbf{Q} \in \mathcal{Q}$, where \mathcal{Q} is defined by

$$\mathcal{Q} \triangleq \{\mathbf{Q} \succeq \mathbf{0}, \text{Tr}(\mathbf{Q}) \leq P\}. \quad (4)$$

III. PRELIMINARIES

The capacity of the compound MIMO channel (1)-(4) is [10, Theorem 7.1]

$$C_{\max \min} = \max_{\mathbf{Q} \in \mathcal{Q}} \min_{\Delta \in \mathcal{E}} I(\mathbf{Q}, \Delta), \quad (5)$$

where $I(\mathbf{Q}, \Delta)$ represents the mutual information of the channel (1) and (2), i.e., [1]

$$I(\mathbf{Q}, \Delta) \triangleq I(\mathbf{x}; \mathbf{y}) = \log \det [\mathbf{I}_r + (\mathbf{H}_0 + \Delta)\mathbf{Q}(\mathbf{H}_0 + \Delta)^H].$$

A dual to (5) is the worst-case channel capacity given by

$$C_{\min \max} = \min_{\Delta \in \mathcal{E}} \max_{\mathbf{Q} \in \mathcal{Q}} I(\mathbf{Q}, \Delta). \quad (6)$$

It is important to distinguish the capacity of compound channel $C_{\max \min}$ and the worst-case channel capacity $C_{\min \max}$: The capacity of compound channel $C_{\max \min}$ in (5) is universally achievable for any channel uncertainty Δ within \mathcal{E} by using the same transmit covariance matrix.¹ The worst-case channel capacity $C_{\min \max}$ is the minimal capacity of the channels with uncertainty $\Delta \in \mathcal{E}$, which requires knowledge of Δ at the transmitter to obtain \mathbf{Q} .² From reliable communication viewpoint, the capacity of compound channel $C_{\max \min}$ is more important than the worst-case channel capacity $C_{\min \max}$, because it requires no knowledge about the channel uncertainty Δ to achieve $C_{\max \min}$. The following relationship is always true

$$C_{\max \min} \leq C_{\min \max}. \quad (7)$$

An example was provided in [10] to illustrate that equality does not always hold in (7). This is not surprising because $C_{\min \max}$ is achieved by using the additional knowledge of Δ , while $C_{\max \min}$ is not.

¹The outer optimization of \mathbf{Q} in (5) is done without knowledge of the realization of Δ .

²The inner optimization of \mathbf{Q} in (6) is done with knowledge of the realization of Δ .

We first recall the closed-form expression of the worst-case channel capacity $C_{\min \max}$ derived in [14]. Let the singular value decomposition (SVD) of \mathbf{H}_0 be $\mathbf{H}_0 = \mathbf{U}_0 \mathbf{\Lambda}_0 \mathbf{V}_0^H$ and $p = \min\{r, t\}$, the following statement is true:

Theorem 1. [14, Theorem 3] *The worst-case channel capacity $C_{\max \min}$ is given by*

$$\begin{aligned} C_{\min \max} &= I(\mathbf{Q}_b, \mathbf{\Delta}_w) \\ &= \sum_{i=1}^p \log [1 + \max\{\sigma_i(\mathbf{H}_0) - \varepsilon, 0\}^2 \lambda_i(\mathbf{Q}_b)], \end{aligned} \quad (8)$$

where the eigen-decomposition of the covariance matrix \mathbf{Q}_b is expressed as

$$\mathbf{Q}_b = \mathbf{V}_0 \mathbf{\Sigma}_b \mathbf{V}_0^H, \quad (9)$$

where the eigenvectors of \mathbf{Q}_b are the right singular vectors of the nominal channel \mathbf{H}_0 , $\mathbf{\Sigma}_b = \text{diag}(\lambda_1(\mathbf{Q}_b), \dots, \lambda_t(\mathbf{Q}_b))$ with the eigenvalues of \mathbf{Q}_b determined by

$$\begin{aligned} \lambda_i(\mathbf{Q}_b) &= \max \left\{ \mu - \frac{1}{\max\{\sigma_i(\mathbf{H}_0) - \varepsilon, 0\}^2}, 0 \right\}, \quad \forall i = 1, \dots, p, \\ \lambda_i(\mathbf{Q}_b) &= 0, \quad \forall i = p+1, \dots, t, \end{aligned} \quad (10)$$

and the water level μ is found from the total power constraint $\sum_{i=1}^p \lambda_i(\mathbf{Q}_b) = P$. The worst-case channel uncertainty $\mathbf{\Delta}_w$ in (8) is given by

$$\mathbf{\Delta}_w = \mathbf{U}_0 \mathbf{\Xi}_w \mathbf{V}_0^H, \quad (11)$$

where $\mathbf{\Xi}_w = \text{diag}(\sigma_1(\mathbf{\Delta}_w), \sigma_2(\mathbf{\Delta}_w), \dots, \sigma_p(\mathbf{\Delta}_w))$ and $\sigma_i(\mathbf{\Delta}_w) = -\min\{\sigma_i(\mathbf{H}_0), \varepsilon\}$. Equation (8) implies that

$$I(\mathbf{Q}, \mathbf{\Delta}_w) \leq I(\mathbf{Q}_b, \mathbf{\Delta}_w), \quad \forall \mathbf{Q} \in \mathcal{Q}. \quad (12)$$

We observe the follows facts from Theorem 1: First, the eigenvectors of \mathbf{Q}_b are the right singular vectors of the nominal channel \mathbf{H}_0 , therefore \mathbf{Q}_b and $\mathbf{H}_0^H \mathbf{H}_0$ are diagonalized simultaneously by the same unitary matrix \mathbf{V}_0 . Second, the worst-case channel perturbation $\mathbf{\Delta}_w$ have the same left and right singular vectors with \mathbf{H}_0 . Finally, the worst-case channel perturbation $\mathbf{\Delta}_w$ in (11) is to reduce the singular values of \mathbf{H} as much as possible, because, according to the singular value inequality [21, Corollary 7.3.8], $\sigma_i(\mathbf{H}) \geq \max\{\sigma_i(\mathbf{H}_0) - \varepsilon, 0\}$, and the singular values of the worst-case channel $\mathbf{H}_w = \mathbf{H}_0 + \mathbf{\Delta}_w$ are exactly given by $\sigma_i(\mathbf{H}_w) = \max\{\sigma_i(\mathbf{H}_0) - \varepsilon, 0\}$.

In [14], the authors showed $C_{\max \min} = C_{\min \max}$ in some special cases, such as high SNR limit, low SNR limit, and rank two nominal channel, etc. However, the capacity of the compound MIMO channel

$C_{\max \min}$ was unknown in general. They conjectured that $C_{\max \min} = C_{\min \max}$ holds in general, not only in these special cases. We prove this conjecture in the next section.

IV. CAPACITY OF COMPOUND MIMO CHANNELS

We first present an interesting saddle point property:

Theorem 2. *The following saddle point property holds*

$$I(\mathbf{Q}, \Delta_w) \leq I(\mathbf{Q}_b, \Delta_w) \leq I(\mathbf{Q}_b, \Delta), \quad \forall \mathbf{Q} \in \mathcal{Q}, \forall \Delta \in \mathcal{E}. \quad (13)$$

where $I(\mathbf{Q}_b, \Delta_w)$, \mathbf{Q}_b , and Δ_w are determined by (8), (9), and (11), respectively.

Since the left inequality in (13) was derived in Theorem 3 of [14] (see (12) of Theorem 1 in last section), we only need to prove the right inequality in (13). However, this is not simple, since $I(\mathbf{Q}_b, \Delta)$ is non-convex in Δ . Similar non-convex problems were solved in some related studies by convex reformulation of the problem [19] and by using singular value inequalities [14]. Unfortunately, these existing methods do not apply to the problem considered here. For this reason, we establish a novel matrix determinant inequality to prove the right inequality in (13). The detailed proof of Theorem 2 is relegated to Section IV-A.

An important corollary of Theorem 2 is provided as follows:

Corollary 1. *The capacity of the compound MIMO channel (1)-(4) is equal to the worst-case channel capacity, i.e.,*

$$C_{\max \min} = C_{\min \max} = I(\mathbf{Q}_b, \Delta_w), \quad (14)$$

where $I(\mathbf{Q}_b, \Delta_w)$, \mathbf{Q}_b , and Δ_w are determined by (8), (9), and (11), respectively.

Corollary 1 follows from a result of von Neumann in 1928 [22, Corollary 9.16], which stated that the saddle point property (13) implies the zero gap relationship (14).

Theorem 2 and Corollary 1 provide the closed-form expressions of the maximum data rate $I(\mathbf{Q}_b, \Delta_w)$ that can be reliably transmitted over the channel with any additive channel uncertainty Δ within \mathcal{E} , and the universal transmit covariance matrix \mathbf{Q}_b that achieves the maximum data rate.

The following remark explains the engineering value of the results in Theorem 2, Corollary 1:

Remark. The results of Theorem 2 and 1 are useful in the design of the adaptive modulation and coding (AMC) schemes for closed-loop MIMO communication systems. In practice, the transmitter usually has

additive and multiplicative channel uncertainties due to channel estimation error, feedback quantization, mobility, etc. The AMC design needs to find the modulation and coding schemes as well as a transmit covariance matrix such that the channel decoding error probability is small and the achieved data rate is not too conservative. If the additive and multiplicative channel uncertainties are known to be in certain bounded regions, we can replace these bounded regions with larger regions expressed by the spectrum norm as those in (3). Then, Theorem 2 and Corollary 1 provide closed-form expressions for the data rate that can be reliably transmitted over the unknown channel and the corresponding universal transmit covariance matrix \mathbf{Q}_b . If \mathbf{Q}_b is feasible for the closed-loop MIMO communication system, one can use \mathbf{Q}_b as the transmit covariance matrix, and select the modulation and coding schemes by considering the rate loss caused by finite length channel codes and practical non-Gaussian discrete modulations.

A. Proof of Theorem 2

We now prove our main result in Theorem 2. First, we need to establish a matrix determinant inequality expressed in Lemma 2, which plays a key role in the proof.

Lemma 1. *Let $\mathbf{\Lambda} = [\varsigma_{ij}] \in \mathbb{R}^{r \times t}$ be a diagonal matrix with non-negative diagonal entries $\varsigma_{ii} \geq 0$ and $\mathbf{\Delta} \in \mathbb{C}^{r \times t}$ be a matrix satisfying $\|\mathbf{\Delta}\|_2 \leq \varepsilon$.*

(1) *The following inequality holds:*

$$|\det(\mathbf{\Lambda} + \mathbf{\Delta})| \geq \prod_{j=1}^p \max\{\varsigma_{jj} - \varepsilon, 0\}, \quad (15)$$

where $p = \min\{t, r\}$.

(2) *Let S be a proper subset of $\{1, 2, \dots, p\}$, then*

$$|\det[\mathbf{\Lambda}(S) + \mathbf{\Delta}(S)]| \geq \prod_{j \in S} \max\{\varsigma_{jj} - \varepsilon, 0\}, \quad \forall S \subseteq \{1, 2, \dots, p\}, \quad (16)$$

where $\mathbf{X}(S)$ is defined in Section I-A, which denotes the submatrix of \mathbf{X} obtained by deleting the rows and columns complementary to those indicated by S from the matrix \mathbf{X} .

(3) *Let $\mathbf{A} = (\mathbf{\Lambda} + \mathbf{\Delta})^H(\mathbf{\Lambda} + \mathbf{\Delta})$, then:*

$$\det[\mathbf{A}(S)] \geq \prod_{j \in S} \max\{\varsigma_{jj} - \varepsilon, 0\}^2, \quad \forall S \subseteq \{1, 2, \dots, p\}. \quad (17)$$

Proof:

(1) It is known that for any $\mathbf{A}, \mathbf{B} \in \mathbb{C}^{r \times t}$ the following singular value inequality holds [23, Eq. (5.12.15)], [21, Corollary 7.3.8]

$$|\sigma_i(\mathbf{A} + \mathbf{B}) - \sigma_i(\mathbf{A})| \leq \|\mathbf{B}\|_2, \quad \forall i = 1, \dots, p.$$

By this, we have

$$\sigma_i(\mathbf{\Lambda} + \mathbf{\Delta}) \geq \max \{ \sigma_i(\mathbf{\Lambda}) - \|\mathbf{\Delta}\|_2, 0 \} \geq \max \{ \sigma_i(\mathbf{\Lambda}) - \varepsilon, 0 \}, \quad \forall i = 1, \dots, p,$$

where the maximization is due to the fact $\sigma_i(\mathbf{\Lambda} + \mathbf{\Delta}) \geq 0$. Since $\varsigma_{ii} \geq 0$, the singular values of the diagonal matrix $\mathbf{\Lambda}$ are given by $\{\varsigma_{11}, \varsigma_{22}, \dots, \varsigma_{pp}\}$. Hence, we attain

$$\begin{aligned} & |\det(\mathbf{\Lambda} + \mathbf{\Delta})| \\ &= \prod_{j=1}^p \sigma_j(\mathbf{\Lambda} + \mathbf{\Delta}) \\ &\geq \prod_{j=1}^p \max \{ \sigma_j(\mathbf{\Lambda}) - \varepsilon, 0 \} \\ &= \prod_{j=1}^p \max \{ \varsigma_{jj} - \varepsilon, 0 \}. \end{aligned}$$

(2) Since $\mathbf{\Lambda}$ is a diagonal matrix, after deleting the rows and columns, the singular values of the submatrix $\mathbf{\Lambda}(S)$ are given by $\{\varsigma_{ii} : i \in S\}$. Moreover, after deleting some rows and columns, the spectrum norm of $\mathbf{\Delta}(S)$ satisfies $\|\mathbf{\Delta}(S)\|_2 \leq \|\mathbf{\Delta}\|_2$ [21, Thorem 7.3.9]. Therefore

$$\begin{aligned} & |\det[\mathbf{\Lambda}(S) + \mathbf{\Delta}(S)]| \\ &= \prod_{j=1}^{|S|} \sigma_j[\mathbf{\Lambda}(S) + \mathbf{\Delta}(S)] \\ &\geq \prod_{j=1}^{|S|} \max \{ \sigma_j(\mathbf{\Lambda}(S)) - \|\mathbf{\Delta}(S)\|_2, 0 \} \\ &\geq \prod_{j=1}^{|S|} \max \{ \sigma_j(\mathbf{\Lambda}(S)) - \varepsilon, 0 \} \\ &= \prod_{j \in S} \max \{ \varsigma_{jj} - \varepsilon, 0 \}. \end{aligned}$$

(3) For any given $S \subseteq \{1, 2, \dots, p\}$, one can interexchange the rows and columns of \mathbf{A} and $(\mathbf{\Lambda} + \mathbf{\Delta})$ by multiplying with two permutation matrices $\mathbf{P} \in \mathbb{S}_+^t$ and $\mathbf{Q} \in \mathbb{S}_+^r$ as $\mathbf{B} = \mathbf{P}\mathbf{A}\mathbf{P}$ and $\mathbf{\Phi} = \mathbf{Q}(\mathbf{\Lambda} + \mathbf{\Delta})\mathbf{P}$, such that $\mathbf{A}(S)$ and $\mathbf{\Lambda}(S) + \mathbf{\Delta}(S)$ are the leading submatrices of \mathbf{B} and $\mathbf{\Phi}$, respectively, i.e.,

$$\mathbf{B} = \begin{pmatrix} \mathbf{A}(S) & \mathbf{C} \\ \mathbf{C}^H & \mathbf{D} \end{pmatrix}, \quad \mathbf{\Phi} = \begin{pmatrix} \mathbf{\Lambda}(S) + \mathbf{\Delta}(S) & \mathbf{M} \\ \mathbf{N} & \mathbf{G} \end{pmatrix},$$

Since $\mathbf{P}^H = \mathbf{P}$, $\mathbf{Q}^H = \mathbf{Q}$, $\mathbf{P}^2 = \mathbf{I}$ and $\mathbf{Q}^2 = \mathbf{I}$, we attain

$$\mathbf{B} = \mathbf{P}(\mathbf{\Lambda} + \mathbf{\Delta})^H \mathbf{Q} \mathbf{Q}(\mathbf{\Lambda} + \mathbf{\Delta}) \mathbf{P} = \mathbf{\Phi}^H \mathbf{\Phi},$$

and thereby

$$\mathbf{A}(S) = (\mathbf{\Lambda}(S) + \mathbf{\Delta}(S))^H (\mathbf{\Lambda}(S) + \mathbf{\Delta}(S)) + \mathbf{N}^H \mathbf{N}.$$

Then, we yield

$$\mathbf{A}(S) \succeq (\mathbf{\Lambda}(S) + \mathbf{\Delta}(S))^H (\mathbf{\Lambda}(S) + \mathbf{\Delta}(S)) \succeq \mathbf{0},$$

which further implies [21, Corollary 7.7.4]

$$\det [\mathbf{A}(S)] \geq \det \left[(\mathbf{\Lambda}(S) + \mathbf{\Delta}(S))^H (\mathbf{\Lambda}(S) + \mathbf{\Delta}(S)) \right].$$

Finally, by using (16), the result in (17) follows. ■

By using part (3) of Lemma 1, we obtain the following matrix determinant inequality:

Lemma 2. (*Matrix Determinant Inequality*) Let $\mathbf{\Lambda} = [\varsigma_{ij}] \in \mathbb{R}^{r \times t}$ be a diagonal matrix with non-negative diagonal entries $\varsigma_{ii} \geq 0$, $\mathbf{D} = [d_{ij}] \in \mathbb{R}^{t \times t}$ be a square diagonal matrix with non-negative diagonal entries $d_{ii} \geq 0$ for $i = 1, \dots, p$ and $d_{(p+1),(p+1)} = \dots = d_{t,t} = 0$ with $p = \min\{t, r\}$, and $\mathbf{\Delta} \in \mathbb{C}^{r \times t}$ be a matrix satisfying $\|\mathbf{\Delta}\|_2 \leq \varepsilon$. Suppose $\mathbf{B} = (\mathbf{\Lambda} + \mathbf{\Delta})^H (\mathbf{\Lambda} + \mathbf{\Delta}) \mathbf{D}$, then

$$\begin{aligned} & \det [\mathbf{I}_p + \mathbf{B}(\{1, 2, \dots, p\})] \\ & \geq \prod_{j=1}^p (1 + \max\{\varsigma_{jj} - \varepsilon, 0\}^2 d_{jj}). \end{aligned} \quad (18)$$

Moreover, we have

$$\begin{aligned} & \det [\mathbf{I}_t + (\mathbf{\Lambda} + \mathbf{\Delta})^H (\mathbf{\Lambda} + \mathbf{\Delta}) \mathbf{D}] \\ & \geq \prod_{j=1}^p (1 + \max\{\varsigma_{jj} - \varepsilon, 0\}^2 d_{jj}). \end{aligned} \quad (19)$$

Proof: We prove (18) by induction. By part (3) of Lemma 1, we attain

$$\det [\mathbf{B}(S)] \geq \prod_{j \in S} (\max\{\varsigma_{jj} - \varepsilon, 0\}^2 d_{jj}), \quad \forall S \subseteq \{1, 2, \dots, p\}. \quad (20)$$

For any set S satisfying $\{1\} \subseteq S \subseteq \{1, 2, \dots, p\}$, by the cofactor (Laplace) expansion [23, Eq. (6.2.5)] of $\det[\mathbf{B}(S)]$ and (20), we attain

$$\begin{aligned} & \det [\mathbf{E}_{11} + \mathbf{B}(S)] \\ & = \det [\mathbf{B}(S)] + \det [\mathbf{B}(S \setminus \{1\})] \\ & \geq \prod_{j \in S} (\max\{\varsigma_{jj} - \varepsilon, 0\}^2 d_{jj}) + \prod_{j \in S \setminus \{1\}} (\max\{\varsigma_{jj} - \varepsilon, 0\}^2 d_{jj}) \\ & = (1 + \max\{\varsigma_{11} - \varepsilon, 0\}^2 d_{11}) \prod_{j \in S \setminus \{1\}} (\max\{\varsigma_{jj} - \varepsilon, 0\}^2 d_{jj}). \end{aligned} \quad (21)$$

Similarly, For any set S satisfying $\{1, 2\} \subseteq S \subseteq \{1, 2, \dots, p\}$, by the cofactor expansion of $\det[\mathbf{E}_{11} + \mathbf{B}(S)]$ and (21), we have

$$\begin{aligned}
& \det [\mathbf{E}_{11} + \mathbf{E}_{22} + \mathbf{B}(S)] \\
&= \det[\mathbf{E}_{11} + \mathbf{B}(S)] + \det[\mathbf{E}_{11} + \mathbf{B}(S \setminus \{2\})] \\
&\geq (1 + \max\{\varsigma_{11} - \varepsilon, 0\}^2 d_{11}) \prod_{j \in S \setminus \{1\}} (\max\{\varsigma_{jj} - \varepsilon, 0\}^2 d_{jj}) \\
&\quad + (1 + \max\{\varsigma_{11} - \varepsilon, 0\}^2 d_{11}) \prod_{j \in S \setminus \{1, 2\}} (\max\{\varsigma_{jj} - \varepsilon, 0\}^2 d_{jj}) \\
&= \prod_{j=1}^2 (1 + \max\{\varsigma_{jj} - \varepsilon, 0\}^2 d_{jj}) \prod_{j \in S \setminus \{1, 2\}} (\max\{\varsigma_{jj} - \varepsilon, 0\}^2 d_{jj}).
\end{aligned}$$

Suppose that for any set S satisfying $\{1, 2, \dots, k\} \subseteq S \subseteq \{1, 2, \dots, p\}$, the following inequalities hold

$$\begin{aligned}
& \det [\mathbf{E}_{11} + \dots + \mathbf{E}_{kk} + \mathbf{B}(S)] \\
&\geq \prod_{j=1}^k (1 + \max\{\varsigma_{jj} - \varepsilon, 0\}^2 d_{jj}) \prod_{j \in S \setminus \{1, 2, \dots, k\}} (\max\{\varsigma_{jj} - \varepsilon, 0\}^2 d_{jj}). \tag{22}
\end{aligned}$$

Then, for any set S satisfying $\{1, 2, \dots, k+1\} \subseteq S \subseteq \{1, 2, \dots, p\}$, by the cofactor expansion of $\det [\mathbf{E}_{11} + \dots + \mathbf{E}_{kk} + \mathbf{B}(S)]$ and (22), we have

$$\begin{aligned}
& \det [\mathbf{E}_{11} + \dots + \mathbf{E}_{(k+1)(k+1)} + \mathbf{B}(S)] \\
&= \det[\mathbf{E}_{11} + \dots + \mathbf{E}_{kk} + \mathbf{B}(S)] \\
&\quad + \det[\mathbf{E}_{11} + \dots + \mathbf{E}_{kk} + \mathbf{B}(S \setminus \{k+1\})] \\
&\geq \prod_{j=1}^k (1 + \max\{\varsigma_{jj} - \varepsilon, 0\}^2 d_{jj}) \prod_{j \in S \setminus \{1, 2, \dots, k\}} (\max\{\varsigma_{jj} - \varepsilon, 0\}^2 d_{jj}) \\
&\quad + \prod_{j=1}^k (1 + \max\{\varsigma_{jj} - \varepsilon, 0\}^2 d_{jj}) \prod_{j \in S \setminus \{1, 2, \dots, k+1\}} (\max\{\varsigma_{jj} - \varepsilon, 0\}^2 d_{jj}) \\
&= \prod_{j=1}^{k+1} (1 + \max\{\varsigma_{jj} - \varepsilon, 0\}^2 d_{jj}) \prod_{j \in S \setminus \{1, 2, \dots, k+1\}} (\max\{\varsigma_{jj} - \varepsilon, 0\}^2 d_{jj})
\end{aligned}$$

By induction, the result of (18) follows.

We now prove (19). According to the determinant identity that $\det(\mathbf{I}_n + \mathbf{F}\mathbf{G}) = \det(\mathbf{I}_m + \mathbf{G}\mathbf{F})$ for any matrices $\mathbf{F} \in \mathbb{C}^{n \times m}$ and $\mathbf{G} \in \mathbb{C}^{m \times n}$, we have

$$\begin{aligned}
& \det [\mathbf{I}_t + (\mathbf{\Lambda} + \mathbf{\Delta})^H (\mathbf{\Lambda} + \mathbf{\Delta}) \mathbf{D}] \\
&= \det [\mathbf{I}_t + \mathbf{D}^{1/2} (\mathbf{\Lambda} + \mathbf{\Delta})^H (\mathbf{\Lambda} + \mathbf{\Delta}) \mathbf{D}^{1/2}].
\end{aligned}$$

Let us define $\mathbf{C} \triangleq \mathbf{D}^{1/2}(\mathbf{\Lambda} + \mathbf{\Delta})^H(\mathbf{\Lambda} + \mathbf{\Delta})\mathbf{D}^{1/2}$. Since $d_{(p+1),(p+1)} = \dots = d_{t,t} = 0$, the j th rows and columns of the matrix \mathbf{C} for all $j > p$ are zero vectors, which implies

$$\mathbf{I}_t + \mathbf{C} = \begin{pmatrix} \mathbf{I}_p + \mathbf{C}(\{1, \dots, p\}) & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{t-p} \end{pmatrix}. \quad (23)$$

According to the definitions of \mathbf{B} and \mathbf{C} , one can readily show that

$$\mathbf{C}(\{1, \dots, p\}) = \mathbf{D}(\{1, \dots, p\})^{1/2} \mathbf{B}(\{1, \dots, p\}) \mathbf{D}(\{1, \dots, p\})^{-1/2}.$$

Hence, we have

$$\begin{aligned} & \det [\mathbf{I}_t + (\mathbf{\Lambda} + \mathbf{\Delta})^H(\mathbf{\Lambda} + \mathbf{\Delta})\mathbf{D}] \\ &= \det [\mathbf{I}_t + \mathbf{B}] \\ &\stackrel{(a)}{=} \det [\mathbf{I}_t + \mathbf{C}] \\ &\stackrel{(b)}{=} \det [\mathbf{I}_p + \mathbf{C}(\{1, \dots, p\})] \\ &\stackrel{(c)}{=} \det [\mathbf{I}_p + \mathbf{B}(\{1, \dots, p\})] \\ &\stackrel{(d)}{\geq} \prod_{j=1}^p (1 + \max\{\zeta_{jj} - \varepsilon, 0\}^2 d_{jj}), \end{aligned}$$

where step (a) and (c) are due to the determinant identity, step (b) is due to (23), and step (d) is due to (18). By this, (19) is proved. \blacksquare

We can now turn to prove Theorem 2.

Proof of Theorem 2:

The left inequality in (13) follows directly from (6) and (8). Therefore, we only need to prove the right inequality in (13). We first reformulate $I(\mathbf{Q}_b, \mathbf{\Delta})$ by making use of (9), i.e.,

$$\begin{aligned} & I(\mathbf{Q}_b, \mathbf{\Delta}) \\ &= \log \det [\mathbf{I}_r + (\mathbf{H}_0 + \mathbf{\Delta})\mathbf{Q}_b(\mathbf{H}_0 + \mathbf{\Delta})^H] \\ &= \log \det [\mathbf{I}_r + (\mathbf{U}_0\mathbf{\Lambda}_0\mathbf{V}_0^H + \mathbf{\Delta})\mathbf{V}_0\mathbf{\Sigma}_b\mathbf{V}_0^H(\mathbf{U}_0\mathbf{\Lambda}_0\mathbf{V}_0^H + \mathbf{\Delta})^H] \\ &= \log \det [\mathbf{I}_r + (\mathbf{\Lambda}_0 + \mathbf{U}_0^H\mathbf{\Delta}\mathbf{V}_0)\mathbf{\Sigma}_b(\mathbf{\Lambda}_0 + \mathbf{U}_0^H\mathbf{\Delta}\mathbf{V}_0)^H] \\ &\stackrel{(a)}{=} \log \det [\mathbf{I}_t + (\mathbf{\Lambda}_0 + \tilde{\mathbf{\Delta}})^H(\mathbf{\Lambda}_0 + \tilde{\mathbf{\Delta}})\mathbf{\Sigma}_b], \end{aligned} \quad (24)$$

where $\tilde{\mathbf{\Delta}} \triangleq \mathbf{U}_0^H\mathbf{\Delta}\mathbf{V}_0$ and step (a) is due to the determinant identity that $\det(\mathbf{I}_n + \mathbf{F}\mathbf{G}) = \det(\mathbf{I}_m + \mathbf{G}\mathbf{F})$

for any matrices $\mathbf{F} \in \mathbb{C}^{n \times m}$ and $\mathbf{G} \in \mathbb{C}^{m \times n}$. Therefore, the right inequality in (13) is equivalent with

$$\begin{aligned} I(\mathbf{Q}_b, \mathbf{\Delta}) &= \log \det \left[\mathbf{I}_t + (\mathbf{\Lambda}_0 + \tilde{\mathbf{\Delta}})^H (\mathbf{\Lambda}_0 + \tilde{\mathbf{\Delta}}) \mathbf{\Sigma}_b \right] \\ &\geq \sum_{i=1}^p \log \left[1 + \max\{\sigma_i(\mathbf{H}_0) - \varepsilon, 0\}^2 \lambda_i(\mathbf{Q}_b) \right], \end{aligned} \quad (25)$$

where $\mathbf{\Lambda}_0$ and $\mathbf{\Sigma}_b$ are both diagonal matrices with non-negative diagonal entries. It is known that $\tilde{\mathbf{\Delta}} \in \mathcal{E}$. Moreover, by (10), the diagonal entries of $\mathbf{\Sigma}_b$ satisfy $\lambda_i(\mathbf{Q}_b) = 0$ for all $i = p + 1, \dots, t$. Then, by Lemma 2, (25) and thereby the right inequality in (13) follows. ■

V. LOWER BOUND FOR THE OUTAGE CAPACITY OF MIMO CHANNELS

It is known that there is a relationship between the capacity of compound MIMO channel and the outage capacity of MIMO channel [19]. In this section, we use this relationship to derive a lower bound for the outage capacity of MIMO channels. The MIMO channel is modeled by $\mathbf{H} = \mathbf{H}_0 + \mathbf{\Delta}$, where the channel uncertainty $\mathbf{\Delta}$ is a random matrix with independent, zero mean, and unit variance complex Guassian distributed entries. The outage capacity of the MIMO channel $C_{out}(\cdot)$ is determined by

$$\begin{aligned} C_{out}(P) &\triangleq \max_{\mathbf{Q} \in \mathcal{Q}, R \in \mathbb{R}} R \\ \text{s.t.} \quad &\Pr\{I(\mathbf{Q}, \mathbf{\Delta}) \geq R\} \geq 1 - P, \end{aligned} \quad (26)$$

where P is the maximum allowable outage probability.

In general, the computation of the outage capacity of MIMO channels is very difficult. In [3], integral based computation method was proposed to compute the outage capacity for Rician MIMO channels with rank one nominal channel. Approximations of the outage capacity for MIMO channels were obtained in [6], [7] for the case of $\mathbf{H}_0 = \mathbf{0}$. To the extent of the authors' knowledge, there is no solution for the outage capacity of MIMO channels for the general cases of arbitrary nominal channel \mathbf{H}_0 . Fortunately, by using the capacity result in Theorem 2, we derive a lower bound of the outage capacity, which is stated in the following lemma:

Lemma 3. *The outage capacity of the MIMO channel $C_{out}(\cdot)$ is lower bounded by the capacity of compound MIMO channel, i.e.,*

$$C_{out}(P) \geq C_{\max \min} = I(\mathbf{Q}_b, \mathbf{\Delta}_w), \quad (27)$$

where \mathbf{Q}_b and $\mathbf{\Delta}_w$ are given by (9) and (11), respectively, and the value of ε in (9) and (11) is given by

$$\varepsilon = \sqrt{F^{-1}(1 - P)}. \quad (28)$$

Here, the function $F(\cdot)$ is determined by

$$F(x) = \frac{\det[\Psi(x)]}{\prod_{k=1}^p \Gamma(p-k+1)\Gamma(q-k+1)},$$

where $p = \min\{t, r\}$, $q = \max\{t, r\}$, $\Psi(x)$ is a $p \times p$ Hankel matrix function of $x \in (0, \infty)$ with entries given by

$$\{\Psi(x)\}_{i,j} = \gamma(q-p+i+j-1, x), \quad i, j = 1, \dots, p,$$

$\Gamma(\cdot)$ and $\gamma(\cdot, \cdot)$ are the gamma function and lower incomplete gamma function, respectively, defined in [24].

Lemma 3 provides an easy-to-compute lower bound for the outage capacity of MIMO channels. We note that this is only a lower bound which is not necessary to be tight. There may be circumstances that the capacity of the compound MIMO channel is zero, while the outage capacity is still positive [19].

It is not difficult to generalize Lemma 3 to the case of joint additive and multiplicative channel uncertainties. The detailed result is omitted here.

Proof: There exist some $\varepsilon > 0$ and \mathcal{E} defined in (3) such that $\Pr\{\mathbf{\Delta} \in \mathcal{E}\} = 1 - P$. According to Theorem 2, for each $\mathbf{\Delta} \in \mathcal{E}$, we have

$$I(\mathbf{Q}_b, \mathbf{\Delta}) \geq I(\mathbf{Q}_b, \mathbf{\Delta}_w).$$

This further implies

$$\Pr\{I(\mathbf{Q}_b, \mathbf{\Delta}) \geq I(\mathbf{Q}_b, \mathbf{\Delta}_w)\} \geq \Pr\{\mathbf{\Delta} \in \mathcal{E}\} = 1 - P.$$

Hence, $R = I(\mathbf{Q}_b, \mathbf{\Delta}_w)$ is feasible for the problem

$$\begin{aligned} R^* &\triangleq \max_{R \in \mathbb{R}} R \\ \text{s.t. } &\Pr\{I(\mathbf{Q}_b, \mathbf{\Delta}) \geq R\} \geq 1 - P, \end{aligned} \tag{29}$$

which indicates $R^* \geq I(\mathbf{Q}_b, \mathbf{\Delta}_w)$. On the other hand, by $\mathbf{Q}_b \in \mathcal{Q}$ and (29), (\mathbf{Q}_b, R^*) is feasible for (26), we have that $C_{out}(P) \geq R^*$. Therefore, (27) follows.

The left task is to determine the value of ε in the expressions of \mathbf{Q}_b and $\mathbf{\Delta}_w$. The value of ε satisfies $\Pr\{\mathbf{\Delta} \in \mathcal{E}\} = 1 - P$. By (3), we have

$$\Pr\{\sigma_1(\mathbf{\Delta}) \leq \varepsilon\} = \Pr\{\mathbf{\Delta} \in \mathcal{E}\} = 1 - P.$$

It is known that the matrix $\mathbf{S} = \mathbf{\Delta}\mathbf{\Delta}^H$ follows a complex Wishart distribution. According to [25], [26], the cumulative distribution function of the largest eigenvalue of \mathbf{S} is given by

$$\Pr\{\lambda_1(S) \leq x\} = F(x).$$

Since $\sigma_1(\mathbf{\Delta})^2 = \lambda_1(S)$ [23, Eq. (7.5.9)], we have

$$\Pr\{\sigma_1(\mathbf{\Delta}) \leq \varepsilon\} = \Pr\{\lambda_1(S) \leq \varepsilon^2\} = F(\varepsilon^2) = 1 - P.$$

Hence, the value of ε is given by (28), and the asserted statement is proved. ■

VI. CONCLUSION

In this paper, we have derived closed-form expression for the capacity of compound MIMO channel with additive uncertainty. We have generalized this result to the case of joint additive and multiplicative channel uncertainties. We have shown that the capacity of compound MIMO channel is equal to the worst-case channel capacity for the considered channel uncertainty model, which establishes a saddle-point property. A lower bound for the outage capacity of MIMO channels has also been derived. These results are helpful for the adaptive transmission designs of closed-loop MIMO communication systems.

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